

The Fill Rate

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Service Level Review

- ▶ Service level: probability that all demand satisfied in a given period
- ▶ We have 99 units in stock and demand was 100:
 - ▶ We did not satisfy all demand: We stocked out by 1 unit
 - ▶ We satisfied (filled) 99% of the demand that period
- ▶ 100 periods, in stock 90 periods (10 periods where we stocked out) \Rightarrow 90% service level
- ▶ The newsvendor model calculates the target service level, so operations people think in terms of that measure
- ▶ Marketing people do not. They think in terms of the *fill rate*

The Fill Rate

- ▶ Demand is 100
- ▶ We have 99 units in stock
- ▶ We satisfy 99% of the demand
- ▶ So the fill rate is 99%
- ▶ If we have a 99% fill rate 3 periods and a 100% fill rate 7 periods, we achieve a service level of 70%

Fill Rate Formula for the Lognormal Distribution

- ▶ Inputs $z = \Phi^{-1}$ (service level) and σ
- ▶ Fill rate $FR = \Phi(z - \sigma) + e^{z\sigma - \frac{\sigma^2}{2}} (1 - \Phi(z))$
- ▶ 3 subparts:
 - ▶ $\Phi(z - \sigma)$
 - ▶ $e^{z\sigma - \frac{\sigma^2}{2}}$
 - ▶ $(1 - \Phi(z))$

Expected Sales and Leftover Inventory

- ▶ Expected sales $\mathbb{E}(S) = \mathbb{E}(D) \times FR$
- ▶ $\mathbb{E}(D) = 1000$ units, $FR = 90\% \Rightarrow \mathbb{E}(S) = 900$ units
- ▶ If we order $Q = 1100$ units, then our expected leftover inventory $\mathbb{E}(LOI) = Q - \mathbb{E}(S) = 1100 - 900 = 200$ units

Calculating Expected Profit as a Multiple of Median Demand for the Lognormal Distribution

- ▶ From σ we calculate $\mathbb{E}(D) = e^{\sigma^2/2}$
- ▶ From overage and underage costs we calculate the newsvendor critical fractile
- ▶ From the critical fractile we calculate the order quantity z in standard deviations $z = \Phi^{-1}(\text{critical fractile})$
- ▶ z and σ give us the order quantity as a multiple of median demand $e^{z\sigma}$ and in units $Q = e^{\mu} e^{z\sigma}$
- ▶ From z and σ we calculate the fill rate FR
- ▶ $\mathbb{E}(S) = \mathbb{E}(D) \times FR$
- ▶ $\mathbb{E}(LOI) = Q - \mathbb{E}(S)$

Expected Profit per Unit of Median Demand

- ▶ Price p , cost c , residual value s
- ▶ Expected profit = $\mathbb{E}(S)(p - c) - \mathbb{E}(LOI)(c - s)$