The Newsvendor Model: Estimating the Profit-Maximizing Order Quantity

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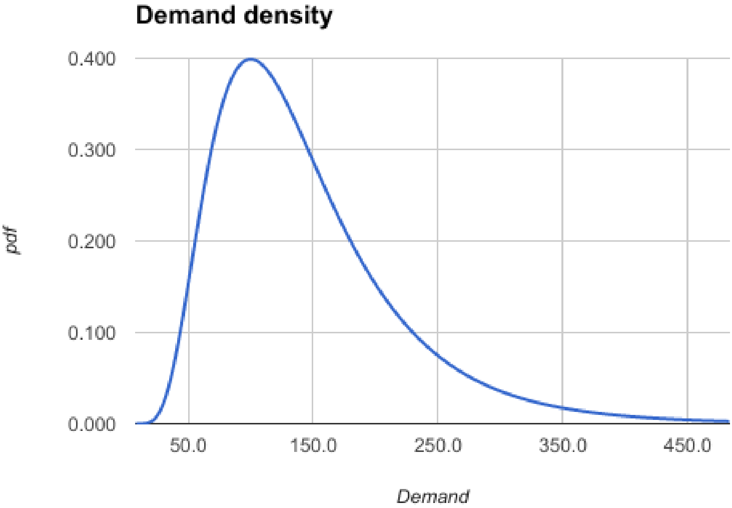
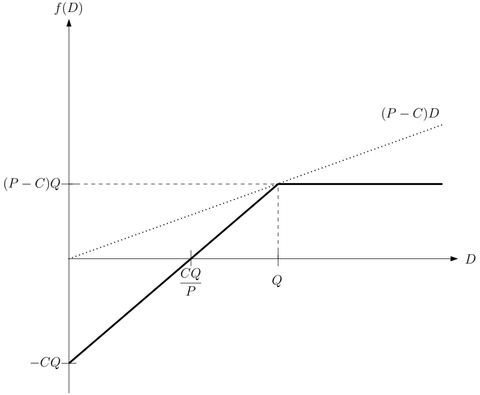
Median demand 100, volatility 0.5



Expected profit $f(D)$ for an order of Q units



- Price P and cost C
- $D > Q \Rightarrow$ profit capped at $(P - C)Q$.
- $D < Q \Rightarrow$ then profit is $(P - C)D - C(Q - D)$

Underage and overage costs

- Price is p, acquisition cost c, and residual value s for an item not sold by the end of the demand period
- Let C_u be the per-unit underage cost. If a stockout results in a lost sale: $C_u = p - c$
- Let C_o be the per-unit overage cost, estimated here as the difference between the cost of a unit and the residual value: $C_o = c - s$

The Newsvendor model

- The newsvendor decides how many newspapers to order for a given day. Too few results in lost sales, but leftover newspapers have no value
- If the order quantity $Q = 0$, then we expect to lose $\mathbb{E}(D)(p - c)$, but incur no overstock cost
- If we increase Q from 0 to 1 unit, then the expected overstock will normally increase slightly, and the expected profit is almost certain to go up by $p - c$
- We keep increasing Q until the expected loss equals the expected gain on the last unit added (the Qth unit), giving us our optimal order quantity $Q^\star$

Should we order Q or $Q+1$ units?

- Define $F(Q)$ as the probability that demand is less than or equal to Q units
- The probability that the $Q+1^{st}$ unit will be left in inventory is $F(Q)$, in which case we will incur C_o
- The probability that the $Q+1^{st}$ unit will be sold is $1-F(Q)$, in which case we will avoid losing C_u
- When the expected gain $(1-F(Q))C_u$ from ordering one additional unit exceeds the expected overstock cost $F(Q)C_o$ then we order the extra unit
- We stop when $F(Q)C_o = (1-F(Q))C_u$
- We solve for $F(Q^*) = \frac{C_u}{C_u+C_o}$, so $Q^* = F^{-1}(\frac{C_u}{C_u+C_o})$

The service level

- The newsvendor model estimates the optimal percentile $F(Q)$ to target, that is, the optimal probability
- This percentile is referred to as the *Service level*
- The profit-maximizing service level is referred to as the *critical fractile* or *critical ratio*
- For normally distributed demand or ln(demand), the critical fractile is interpreted as $\Phi(z)$ rather than $F(Q)$
- Critical fractile $\Rightarrow$ number of standard deviations $\Rightarrow$ order quantity

Residual value

- The residual value is always less than the acquisition cost: $s < c$
- The residual value can be negative if there is a disposal cost
- Another term used for the residual value is *salvage value*, especially for products that are liquidated below cost

Example

- Price $p = \$100$
- Per-unit acquisition cost $c = \$44$
- Residual value $s = \$20$
- $C_u = 100 - 44 = 56$
- $C_o = 44 - 20 = 24$
- $\Phi(z^*) = \frac{56}{56+24} = 0.7$
- $z^* = \Phi^{-1}(0.7) = 0.52$ standard deviations
- $Q^* =$ Median demand $\times\, e^{0.52\sigma}$ for demand volatility σ
- Let $\sigma = 0.4 \Rightarrow$ the optimal order quantity is $e^{0.52\times 0.4} = 1.2$ times median demand

Second order opportunity

- Now consider a responsive supplier who lets us order extra units if $D > Q$, but charges a higher cost per unit (e.g., 60 rather than 44, so a premium of 16)
- The cost of stocking out C_u decreases from the full loss of profit to that cost premium
- The cost of overstock C_o doesn't change
- The revised critical fractile is $\frac{16}{16+24} = 0.4 <$ median demand
- $\Phi^{-1}(0.4) = -0.25 \Rightarrow Q^* = e^{-0.25*0.4} \approx 0.9\times$ median demand

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