Randomness of Demand

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Gestion des opérations

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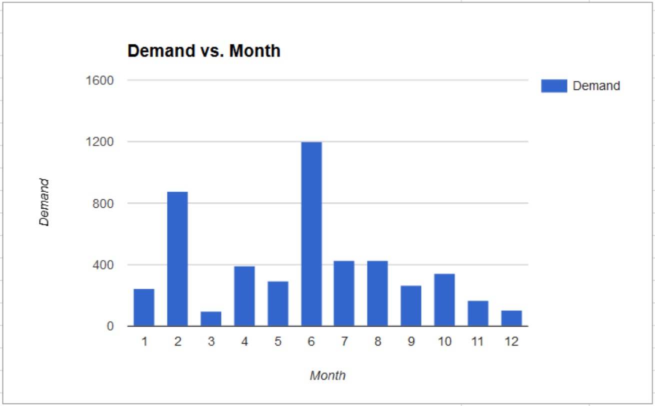
Placing an order before knowing demand:

- Demand is random
- If we order too little we will stock out (loss of profit and customer good will)
- If we order too much we will have to pay inventory-holding costs, sell the excess at a loss, or discard the excess

Demand data

https://goo.gl/ZKHyPA


Demand vs. Month
1600
1200
800
400
0
Demand
1 2 3 4 5 6 7 8 9 10 11 12
Month
Demand

Some useful values to calculate:

- Mean 403.8
- Standard deviation 323.8
- Coefficient of variation $\frac{\text{standard deviation}}{\text{mean}} = 0.8$

How plausible is the usual assumption that demand is normally distributed?

- If normal, then 99.7% of the time, demand should fall within 3 standard deviations of the mean
- Maximum: $403.8 + 3 \times 323.8 = 1375.6$
- Minimum: $403.8 - 3 \times 323.8 = -567.9$
- Negative demand is not plausible...

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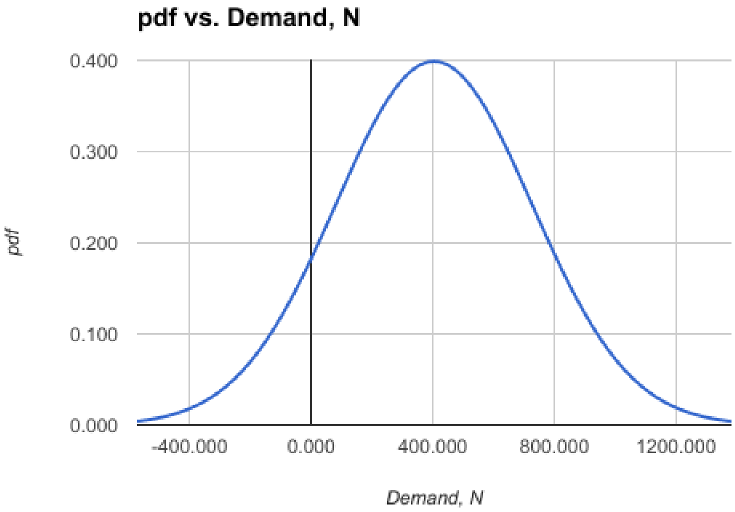
pdf vs. Demand, N
0.400
0.300
0.200
0.100
0.000
pdf
-400.000
0.000
400.000
800.000
1200.000
Demand, N

Comparing the mean to the median

- Median ("geometric mean") lies at the 50th percentile of the demand density: There is a 50% chance that demand will be less than the median, and a 50% chance that it will be higher
- If the median = (arithmetic) mean $\Rightarrow$ symmetric distribution (e.g., normal)
- Median $<$ mean $\Rightarrow$ heavy right tail
- $\Rightarrow$ lognormal density a plausible choice

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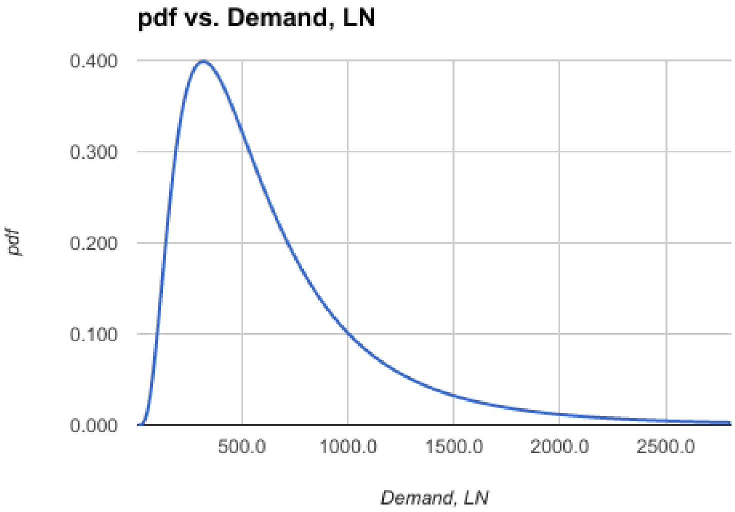
Let's check our data:

- Mean 403.8
- Median 318
- Coefficient of variation greater than 0.33, so more than 3 standard deviations creates a problem with negative demand
- Lognormal more plausible than normal


pdf vs. Demand, LN
0.400
0.300
0.200
0.100
0.000
pdf
500.0
1000.0
1500.0
2000.0
2500.0
Demand, LN

Ways to estimate the volatility of demand

- Take the standard deviation of ln(demand) values (if you have data)
- Use the ratio of the mean to the median
- Use the coefficient of variation
- Use intuition about demand peaks

Estimating the volatility from the mean and the median

- $\ln(\text{demand}) \sim \mathcal{N}(\tau, \sigma^2)$
- σ (the standard deviation of ln demand) is referred to as the "volatility" of demand
- Median demand $= e^{\tau}$
- Expected demand $= e^{\tau+\sigma^2/2} = e^{\tau} e^{\sigma^2/2}$
- We can estimate the volatility from the ratio of expected to median demand
- $\frac{403.8}{318} = 1.27 = e^{\sigma^2/2}$
- $\sigma = \sqrt{2\ln(1.27)} \approx 0.7$

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Estimating volatility from the coefficient of variation

- We can obtain σ from the coefficient of variation using the formula $CV = \sqrt{e^{\sigma^2} - 1}$
- Thus, $\sigma = \sqrt{\ln(CV^2 + 1)}$
- For our 12 data points above with a CV of 0.8, we estimate that the volatility of demand is approximately $\sqrt{\ln(0.8^2 + 1)} = 0.7$

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Standard deviations and percentiles

- With the normal distribution, if we know how many standard deviations we are from the mean, we know the percentile
- What is the 80th percentile of our demand density?
- $\Phi^{-1}(0.80) = 0.84$ standard deviations: If demand is normally distributed and we have 0.84 standard deviations available above the mean, there is an 80% chance that we will satisfy all demand.
- We will also be able to make use of this principle for the lognormal distribution, because ln(demand) is normally distributed.

Estimating volatility from the peak-to-median ratio

- Peak demand over the 12 months was 1200
- Manager: "We normally see a demand value that high or higher about once every 3 years"
- $\Rightarrow$ 1 month out of 36
- The chance of demand being lower than 1200 is $\frac{35}{36} \approx 97.2\%$
- 97% corresponds to about 1.9 standard deviations
- $1200 = \text{Median} \times e^{1.9\sigma}$
- The ratio of peak to median demand is $e^{1.9\sigma}$
- $\ln(\frac{\text{peak}}{\text{median}})/1.9 = \sigma$
- $\ln(\frac{1200}{318})/1.9 \approx 0.7$

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What other densities do we need to consider?

- If demand follows a normal distribution, then the lognormal will be about the same
- If demand has a heavier tail than the lognormal, the lognormal will still give us a very good idea
- Start by assuming a lognormal and you will probably be fine

Essential concepts

- Randomness of demand
- Calculation of the mean, median, standard deviation, coefficient of variation
- Compare the mean to the median to see whether it is safer to assume that demand is normal or lognormal (lognormal is always ok, but some managers prefer the normal)
- 4 ways to estimate the volatility
- Getting from standard deviations to percentiles and back (using a table or using Excel)